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Gas Turbine Blade Reliability and Generator Optimal Estimation of Weilbull Probability Distribution

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ABSTRACT

The increasing demand of electricity power supply and its availability is indispensable to manage economically following the rapid growth of population to energy consumption capacity which is a major concern making the electricity sector industries to experienced significant implication for power plant operation to provide basic energy services to the people particularly to the reliability of the engineering component under review To achieve this goal the study will consider the application of reliability technique in order to analyse the activities of the failure times of ten identical gas-turbine blade of similar "make" and mode of operations subjected to the same conditions in Afam power station over a period of ten years, from the results obtained, the gas-turbine blades were in their wear-out period of reliable maintenance to give out efficient performance following to the reliability of the three parameter weilbull distribution R(t) given as 0.60 this means that turbine blade actually required reliable maintenance. While, the failure rate of the turbine-blade $\lambda(t): 0.082577/hrs$, *the mean time to failure (MTTF): 13.55hours. The results obtained through the simulation of TPC windchill quality solution software estimated the parameters which shows suitable behaviour of the system components for early response for reliable maintenance. The reliability* $R(t)$, *failure rate* $\lambda(t)$ *and meantime to failure (MTTF) were successively computed. Conclusively, the probability that the gas turbine blades under investigation will continue to be operational in service without failures is about 70% while the mean time to failure of the gas turbine blade is about 14hours.*

KEYWORDS: Availability, Reliability, Electric Power, Weilbull Distribution.

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1. INTRODUCTION

Gas turbine-blade component reliability plays a significant role in the performance of electricity power supply of power from the generating station which incidentally falls among the expensive equipment in the electricity power supply industry. Therefore, it is a necessity to guide and reduce the risk of failures and forced outages by system design engineer technique on the view to perform preventive repairs to improve the existing reliability of the system (Kang *et al.*, 2018). The gas turbine blade wear-out of failure has leads to excessive cost of electricity outages and customer dissatisfaction. Thus, evaluation and analysis of the behaviour of this facilities/equipment requires strong knowledge about regular conditions of data collection to provide adequate information for effective system planning and operations. (Chen *et al.*, 2016)

The aim of this paper is to analyse the Gas turbine blade reliability and generator optimal estimation of the weilbull probability distribution. Energy utilization is the fundamental element for economic and social growth, particularly in Nigeria.

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The migration of people from the rural setting to urban cities as a form of socio-economic development constitute an overload in the network thereby charcterizing the process as a form of complex and robust challenging problems to solve mathematically. Power system comprises generation transmission and distribution network. Electric power is generated and distributed upto the point of utilization (consumer). In Nigeria, the activities of transmission and distribution sector are characterized by constant outages due to system components failures as an interruption to the efficient power supply reliability. The cost of repairs, equipments down time, idle labour, loss of output schedule delays and customer dissatisfaction are thereby affected in small, middle, and large economic business operations.

The main objective of engaging preventive maintenance is to reduce the total cost of providing services. The cost analysis between breakdown and preventive maintenance can indicate the preferred alternative. To conduct a cost-analysis an information must be available relative to:

(i) Cost of breakdown
(ii) Frequency of breakdown
(iii) Cost of preventive maintenance to reduce or eliminate failures (otherwise preventive maintenance mean routine inspection and servicing).

However, it is designed to detect potential failure conditions and make correction that will prevent major operating difficulties. It is important to effect quick responses to machine service requirements that are known whose failures can be predicted with some level of accuracy. Preventive maintenance is desirable when it can increase the operating time of asset by reducing the severity and frequency of breakdown. Preventive maintenance might include cleaning, lubricating, inspecting, calibration, testing, critical part placement before failure or complete over-hauls.

2.0 MATERIALS AND METHOD

2.1 Materials

The materials used in this research include:

(i) Data of the facilities (maintenance records)
(ii) Data consists of times(t) to failure of ten identical gas turbine blades in the study case (Afam power plant)
(iii) Application of TPC windchil quality solutions software tool.

These data were collected from the Port Harcourt Electricity Distribution Company (PHEDC/Research Desk and operation, Maintenance Department). The data collected was implemented into governing equations to get information of the turbine blade failures to predicts/estimate the plan to reduce early failure rate (Balakrishnan & Kateri, 2008).

2.2 Methodology

A reliability research strategy was utilized for this review following to the reliability model described by three parameter estimation of gas-turbine blade performance which can examine the failure times of gas turbine blade for ten similar units, under investigation and evaluation.

The methodology was based on the following considerations:

(i) Collecting the frequency of turbine blade failure that results into outages and blackout
(ii) Determination of mean time to failure (MTTF) and failure rate
(iii) Determination of reliability function of the three parameter weilbull distribution.
(iv) Determination of unreliability that may either result into availability and unavailability.
(v) Examination of life cycle of the system components for efficient planning.

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2.3 Computation of the characterised reliability-index and three parameter weilbull distribution

Case 1: The Mean time to failure of the three-parameter weilbull distribution (MTTF) is given by:

$$\text{MTTF} = \gamma + \eta\, \Gamma\left(1+\frac{1}{\beta}\right) \qquad (1)$$

where.

$$\left.\begin{array}{l} \gamma \geq 0 \text{ and } t,\ \beta,\ \eta > 0 \\ \Gamma(x) = \int_0^\infty e^{-x} x^{n-1} \\ \eta \ : \text{Scale parameter} \end{array}\right\} \qquad (2)$$

$\Gamma(x)$: Gamma Function

γ : Location parameter

β : shape parameter

t : Time

That is the term (MTTF) is applied to non-repairable points which operates under specified condition. It is otherwise the ratio of sum of time to failure of each component to the number of components under test.

Case 2: Mean time between failure (MTBF)

This is the term, which is applied to repairable terms, which measure the average time that a particular equipment will fail and remain in service. MTBF of an equipment may be reduced to potential defects introduced by poor maintenance procedures.

Thus,

$$\text{MTBF} = \frac{1}{n}\sum_{i=1}^{n}(t_k \ldots t_{k-1}) = \frac{t_n - t_o}{n} = \frac{t_n}{n} \qquad (3)$$

Since $t_0 = 0$ at the beginning.

Then,

$$\text{MTBF} = \frac{Total\ operating\ time}{No.\ of\ \ failures\ in\ that\ time} \qquad \text{`}(4)$$

Case 3: Availability performance

Availability performance is the ability of an item to be in a state to perform a required function under a given conditions that is for a given instance of time or over a given time interval, this means that;

(i) All items assumed operating conditions unless failed scenario.

(ii) The exception would have been standby redundancy but this scarcly exists power station because of high power supply demand.

(iii) The outcomes in the analysis are based on two fundamental rules for combining probabilities.

(iv) If A and B are two independent events with probabilities $\rho(A)$ and $\rho(B)$ of occurring, then the probability $\rho(AB)$ that both events will occur is the product.

$$\rho(AB) = \rho(A)\,.\,\rho(B) \qquad (5)$$

(v) Similarly, if two events A and B are mutually exclusive so that when one occurs the other cannot occur, the probability that either A or B will occur is :

$$p(A\,B) = p(A) + p(B) \qquad (6)$$

(Datsiou & Overend, 2018)

Case 4: Failure rate

Failure may be either partial or complete, gradual, or sudden it may be caused by inherent weaknesses or misuse. These failures can manifest in the following forms as, catastrophic failures, primary failure, and secondary failures.

Therefore, failure-rate is related to both number of failures per unit time that is the number of items which fails in each time depends not only on the quality of the item, hence.

(i) If the number of components in operation at the time of failure is Nr
Then failure –rate $\lambda(t)$ is given by.

$$\lambda(t) = \underset{\Delta t \to \infty}{Lim} \frac{1}{N_s} \times \frac{\Delta N_f}{\Delta_t} = \frac{1}{N_s} \times \frac{\delta N_f}{\delta t} \qquad (7)$$

Case 5: Operational availability

The operational availability (A_0) given as.

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$$A_o = \frac{Up - Time}{Operating - Time} \qquad (8)$$

Thus,

Availability,

$$(A_V) = \frac{Available\ Hour}{Period\ Hour} \times \frac{100}{1} \qquad (9)$$

The significant measurement of the performance of a repairable system given as.

$$R(t) = \ell^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}, t \geq \gamma \qquad (10)$$

Case 6: Reliability function of the three (3) – parameter weilbull distribution is given as.

The three –parameters weilbull failure rate function is given by.

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1}, t \geq \gamma \qquad (11)$$

Case 7: Weilbull shape parameter(β)

The weilbull shape parameter (β) is also known as the weilbull slope. This is because the value of β is equal to the slope of a line in a probability plot.

(i) When the shape parameter , $\beta < 1$ (this means that the failure rate decreases)

(ii) When the shape parameter, β = 1 (this means that failure-rate is constant with time (t) and the distribution is equal to the exponential distribution)

(iii) When the shape parameter, β > 1 (this means that failure rate increases)

Case 8: Weilbull scale parameter, η

That is increasing the value of η while keeping β constant has the effects of stretching out the probability density function (pdf). A change in the scale parameter (η) has the same effect on the distribution as a change of the abscissa scale. Since the area under a pdf curve is a constant value, the peak of the pdf curves will also decrease with increase of η (Yang & Nie, 2007).

Case 9: Weilbull Location Parameter, γ

The location parameter, γ accounts for the substraction (positive or negative) value that places the points in an acceptable straight line. changing the value of the location parameter γ , has the effects of pushing the distribution and associated function if (γ > 0) or to the left if (γ < 0).

Case 10: Prediction Performance of Weilbull Distribution Model

The prediction accuracy of the model in the estimation of the turbine-blade failures with respect to actual values were evaluated based on the correlation coefficient R^2, root mean square error (RMSE) and coefficient of efficiency (COE) (Markovic *et al.*, 2009). These parameters are calculated based on the following equation as.

$$R^2 = \frac{\sum_{i=1}^{N}(y_i - z)^2 - \sum_{i=1}^{N}(x_i - z)^2}{\sum_{i=1}^{N}(y_i - z)^2} \qquad (12)$$

Similarly, the root means square error (RMSE) given as.

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}(y_i - x_i)^2\right]^{1/2} \qquad (13)$$

The coefficient of efficiency (COE) given as.

$$COE = \frac{\sum_{i=1}^{N}(y_i - x_i)^2}{\sum_{i=1}^{N}(y_i - z)^2 -} \qquad (14)$$

where y_i is the i^{th} actual data X_i is the i^{th} predicted data with the weilbull distribution (*z*) is the mean of the actual data, *N* is the number of observations.

Case 11: Reliability model of system component and weilbull-two parameter characterization.

The rate of failure and mean time between failure (MTBF) are the key parameters of reliability in the turbine blades evaluation which are estimated using weilbull distribution function and with available data for each part, since weilbull distribution technique is a vital tool used in the systematic modeling of failure rates, forecasting failures and in modeling of failure and fault-process stemmed from their aging.

The weilbull parameter would be determined using the least-square relationship as:

$$y_i = mx_i + c \ \ or \ \ x_i = In(t_i) \quad (15)$$

where, t_i is the independent age (year) of failed component in rank i.

Therefore,

$$y_i = In\ In\left[\frac{1}{1-F_i}\right] \quad (16)$$

From, (15) and (16) are the weilbull shape parameter (β) which can be calculated given as.

$$\beta = m = \frac{\sum_{i=1}^{N} x_i y_i - \frac{\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N}}{\sum_{i=1}^{N} x_i^2 - \frac{\left[\sum_{i=1}^{N} x_i\right]^2}{N}} \quad (17)$$

Similarly, the constant (c) given as.

$$C = \frac{\sum_{i=1}^{N} y_i}{N} - m\frac{\sum_{i=1}^{N} x_i}{N} \quad (18)$$

The life or scale parameter (α) can be determined given as.

$$\alpha = \ell^{-\left[\frac{c}{m}\right]} \quad (19)$$

(i) By the estimation of two-parameters technique for the prediction if the system behavior of the component is according to the equipment – curve distribution.

(ii) Weilbull probability distribution function f(t) shows probability of failure in certain time (t) given as.

$$F(t) = \left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1} \ell^{-\left(\frac{t}{\alpha}\right)^{\beta}}; \ for \begin{cases} \alpha > 0 \\ \beta > 0 \\ 0 \le t <= \infty \end{cases} \quad (20)$$

(iii) The cumulative distribution function F(t), which shows the probability of failure in time (t) would be calculated as.

$$F(t) = 1 - \ell^{-\frac{t}{\alpha}^{\beta}}; \ for \begin{cases} \alpha > 0 \\ \beta > 0 \\ 0 \le t <= \infty \end{cases} \quad (21)$$

Reliability function R(t) which shows probability of remaining intact till the time (t) and the rates of failure λ(t) which can be expressed as.

$$R(t) = 1 - F(t) = \ell - \left(\frac{t}{\alpha}\right)^{\beta} \ or \quad (22)$$

$$\lambda(t) = \frac{F(t)}{R(t)} = \left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1} \quad (23)$$

From observation, the shape –parameter (β) affects the shape distribution curve that is when the shape parameter changed, the curve f(t) varies differently in shape. For example, if the curve turns to exponential distribution while β = 1

(i) That is the failure rate will be decreasing while β<1, means that the component is in the early failure state.

(ii) Similarly, when failure rate is constant while, β=1, the components is in the occasional failure condition. Incidentally, the failure rate is increasing while β>1 this means that the component is in the loss failure condition (Abbasi & Malik, 2016).

Case 12: The Mean and standard deviation of a weilbull distribution evaluation

The mean and standard deviations are presented in terms of shape and scale –parameter given as:

$$\mu = \mu\Gamma\left(1 + \frac{1}{\beta}\right) \quad (24)$$

and

$$\delta^2 = \alpha^2\left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right] \quad (25)$$

2.4 Collection of Failure Time (t) of a Gas Turbine-Blades performance

Presentation of the failure time (hrs) of a ten identical gas turbine–blades that are similar-make and similar –model while working under same stress conditions, stated as.

Table 1: The rating –order of turbine blade failures in time (hrs)

Rank	Failure (hrs)	Time,
1	1047	
2	1279	
3	1340	
4	1578	
5	1598	
6	1749	
7	1804	
8	1841	
9	1847	
10	1869	
11	1879	
12	1890	
13	1939	
14	1948	
15	1949	
16	1956	
17	19087	
18	1995	
19	2004	
20	2005	
21	2047	
22	2214	
23	2287	
24	2436	
25	2439	
26	2442	
27	2581	
28	2617	
29	2926	
30	2978	

Sources: Research Desk/Maintenance Department Afam power Generating Station.

3.0 RESULTS AND DISCUSSION

3.1 Results

Table 1 shows that failure times (hours) of ten similar gas turbine blades performance reliability having identical configuration in the make and model when subjected to the same stress level conditions under investigations were determined. The existing failures data is analyzed using weilbull analysis software TPC windchill Quality solutions to obtain three-parameter weilbull estimates of location, shape, and scale parameters in order to produce the following weilbull probability plot, probability density function (PDF) - plot, and 3D contour plot as presented in table 2, given as:

Table 2: Three (3) – parameter weilbull estimates using windchill quality solutions application tool

Function	Shape – parameter (β)	Scale – parameter (η)	Location parameter (λ)
Probability	4.1652	33.6733	–3.1855
Reliability with time	4.1652	33.6733	–3.1855
Unreliability with time	4.1652	33.6733	–3.1855

The application of reliability evaluation of three-parameter weilbull distribution, shows from the simulation results obtained in the TPC windchill software reliability application tool given as:

shape (β) = 4.1652

scale (η) = 33.6733

location (λ) = –13.1855

time (t) =16.1867 hours

Thus, the reliability function of the three-parameter weilbull distribution given as:

$$R(t) = \ell - \left(\frac{t-\lambda}{\eta}\right)^{\beta}, \text{ for } t \geq \gamma \qquad (26)$$

Similarly,

The three –parameter weilbull failures rate function is given by:

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t-\lambda}{\eta}\right)^{\beta-1}, \text{ for } t \geq \gamma \qquad (27)$$

and

Mean-time to failure of the three-parameter weilbull distribution is given as:

$$MTTF = \gamma + \eta + \left(1 + \frac{1}{\beta}\right), \text{ where } \gamma \geq 0 \text{ and } t, \beta, \eta > 0$$

that is,

$$\Gamma(x) = \int_0^\infty \ell^{-x} x^{n-1} \qquad (28)$$

where.

η : scale parameter

$\Gamma(x)$: gamma function

γ : location parameter

β : shape parameter

t : time

Thus, the reliability analysis for the weilbull distribution parameters is determined and calculated as.

$$R(t) = \ell - \left(\frac{t-\gamma}{\eta}\right)^{\beta},$$

t = 16.1867hours

ℓ = 2.718

β = 4.1652

η = 33.6733

γ = –13.1855

$$R(t) = \ell^{-}\left(\frac{16.1867 - 13.1856}{33.6733}\right)^{4.1652}$$

or

$$R(t) = \ell^{-}\left(\frac{29.3723}{33.6733}\right)^{4.1652}$$

or

$$R(t) = \ell^{-}(0.872272)^{4.1652}$$

or

$$R(t) = \ell^{-0.56598} = 2.718^{-0.56598}$$

or

$$R(t) = \frac{1}{2.718^{0.56598}} = \frac{1}{1.7610695}$$

$$R(t) = 0.5678 \approx 0.60$$

Similarly, the failure rate is calculated as:

$$R(t) = \frac{\beta^{-}}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} \quad for\ t \geq \gamma$$

β = 4.1652, t = 16.1867hrs

η = 33.6733 γ = 13.1855hrs

$$\lambda(t) = \frac{4.1652}{33.6733}\left(\frac{16.1867 - 13.1855}{3.6733}\right)^{4.1652}$$

or

$$\lambda(t) = 0.12369\left(\frac{29.3723}{33.6733}\right)^{3.1652}$$

or

$$\lambda(t) = 0.12369(0.872272)^{3.1652}$$

or

$$\lambda(t) = 0.12369 \times (0.648862)$$

or

$$\lambda(t) = 0.0802577\ /\ hours$$

Thus, the Mean time to failure (MTTF) of the three-parameter weilbul distribution calculated as.

$$MTTF = \lambda + \eta\ \Gamma\left(1 + \frac{1}{\beta}\right),\ for\ \gamma \geq 0\ and\ t, \beta, \eta > 0$$

γ = –13.1855, η = 33.6733

β = 4.1652, Γ 1.2400 = 0.7940

$$MTTF = -13.1855 + 33.6733\ \Gamma\left(1 + \frac{1}{4.1652}\right) \text{ or}$$

$$= +13.551102\, hrs$$

3.2 Discussion of Weilbull Distribution Plot

Following to the presentation plot of figure 1 the probability of failure versus time are shown. The scope of the probability plot is seen to be decreasing at the beginning but as it gets to the end of the plot it is observed to be gradually increasing which is pushed to the right. Provided the value of the location parameter is positive.

Similarly, in figure 2, shows the plot of the probability density function (pdf) observed to be increasing at steady condition up to certain time, t =18hours but from this point further there is a sharp decrease. Subsequently, in Figure 3 which shows the reliability plot, it is observed to be initially extremely high at the beginning but over time becomes decreasing, which means that the gas turbine is aging with time that is there is a gradual drop in reliability. Similarly, figure 4, shows the plots of the failure rate which is increasing at a time (t). This indicates that gas turbine blade is tilting towards their wear-out period of reliable maintenance.

Figure 5 shows vivid presentation of the contour plot of 3D, from the plots it is observed that the values of the estimates of three-parameter weibull vary along the contour axes. Significantly, that is moving from the bottom of the location and shape parameters which increases gradually while the value of the scale-parameter observed to be slightly affected.

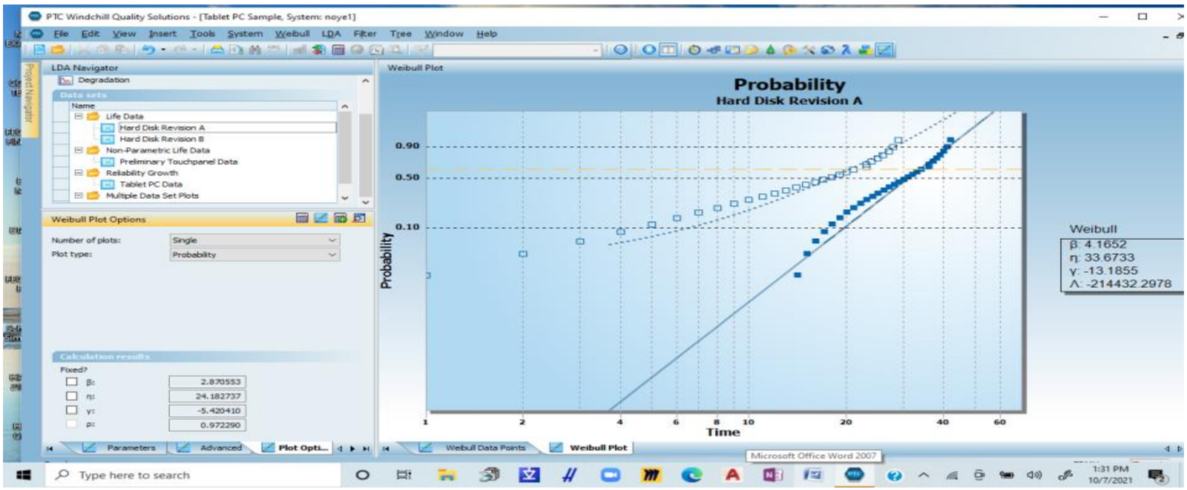

Figure 1: Probability plot against time, t in hours

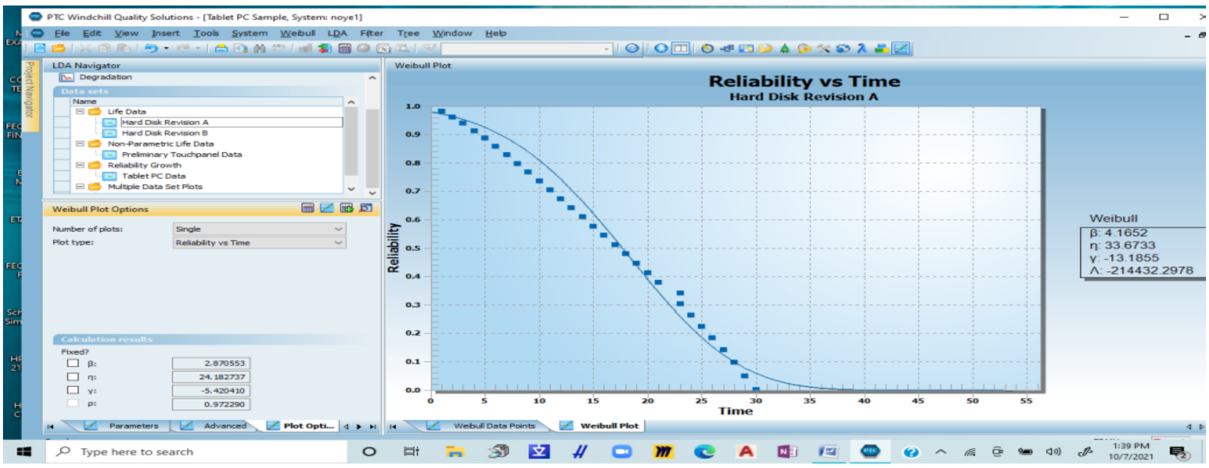

Figure 2: Reliability versus time in hours

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Figure 3: Unreliability versus time, (t) in hours

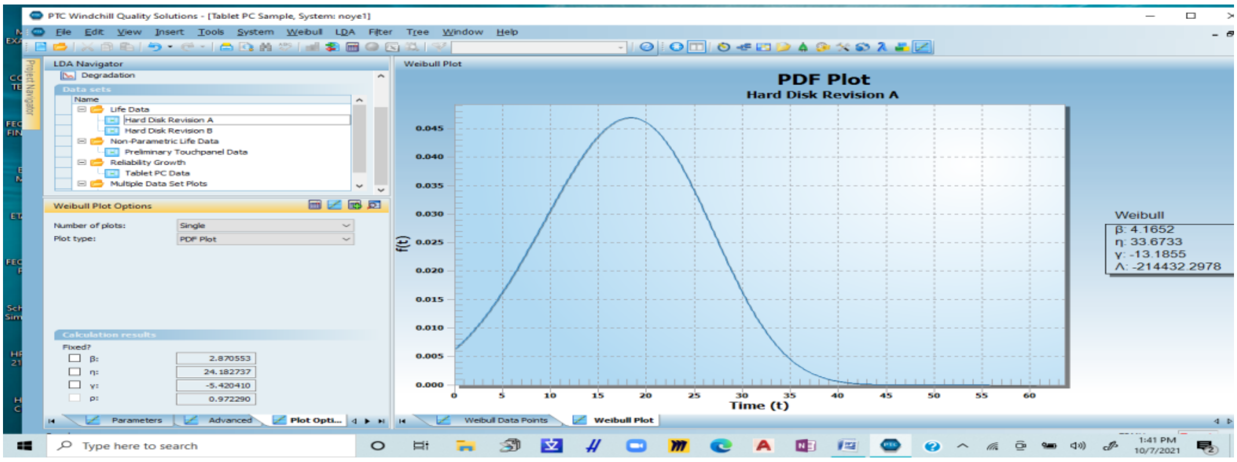

Figure 4: Probability density function (PDF), versus time (t) in hours

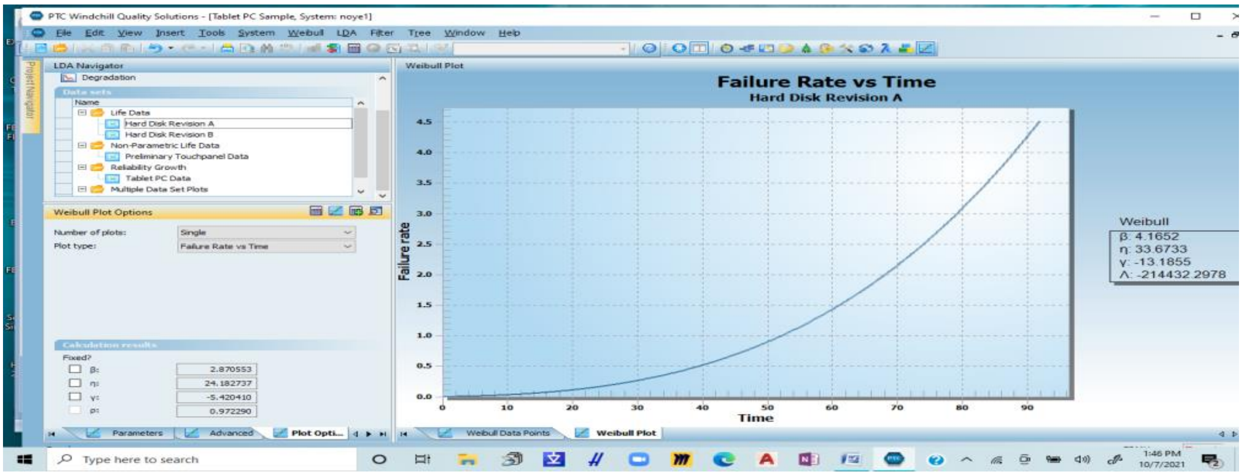

Figure 5: Failure rate versus time, t in hours

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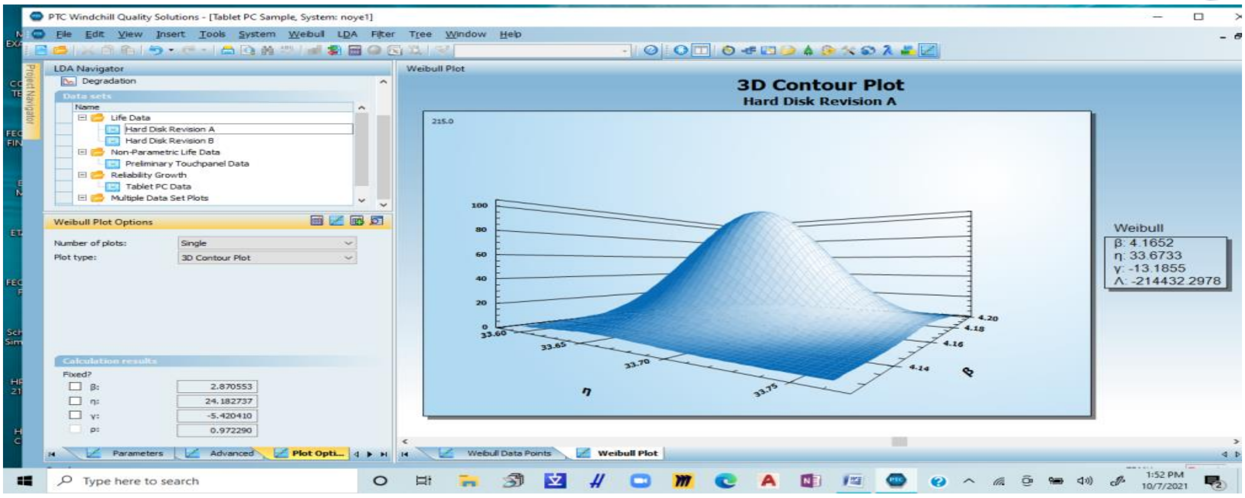

Figure 6: 3D Contour representation plot

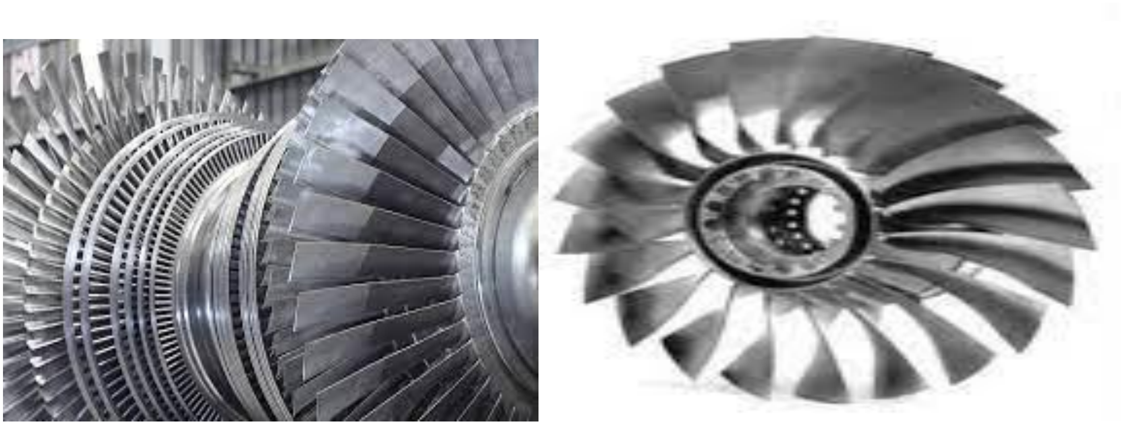

Plate 1: Turbine-blade design configuration.

4.0 CONCLUSION

The analysis of the parameter estimation of weilbul distributions using reliability evaluation technique was formulated with implementation of the imput data collected in the study case under investigation. The study evaluated the analysis of the failure times of ten similar turbine blade in an identical mode of operations using TPC windchill quality solution software to obtain the estimates of the three parameters of weillbull (location, shape, and scale parameters). The estimates were successfully obtained which represents the probability that gas-turbine blade existing state will continue to be operational in service without components failure which is about 70% reliability level, while the meantime to failure to the turbine blades is about 14hours.

The results of the analysis evaluated indicates that the blades were tilting towards their wear-out period of their numerical value of shape-parameter (β) which becomes 4.1652 that is greater than one. From the TPC windchill estimates obtained the reliability $R(t)$, failure rate $\lambda(t)$ and mean-time to failure of the blades are computed.

This research paper is a lead study and a challenging area in widespread practice especially in management of system components of power plant unit availability that should conform to local and international standard of operations. The following recommendation are strongly addressed to improve the existing reliability of power quality as:

(i) The electric power station should align in the development of equipment/components for specific operations and maintenance (O&M) procedures and program.
(ii) Energy demand and load growth should be monitored from the station, based on the following subsequent demand rate and frequency.
(iii) Electric power station should embrace the ideas or the use of dedicated high-profile software – package for analysis, estimate and evaluations.
(iv) Reliability indices and parameters analysis should give self-contained information to give a useful practical introduction to standard availability performance evaluations.
(v) Wear-out period of system components (turbine blade) should be replaced and service immediately to avoid total breakdown of other facilities that may affects or attracts more cost to the machine (turbine power plant) etc.

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Nomenclature & Units

η : Scale parameter

$\Gamma(x)$: Gamma Function

γ : Location parameter

β : shape parameter

t : Time