



Maximization of Output and Effective Load Dispatch of Afam Power Plant

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ABSTRACT

This paper presents the outcome of our research on the optimization of output in the present-day Nigerian power stations using Afam power plant as a case study. Afam power plant is a representative of the about 23 power generating plants connected to the Nigerian national grid which has a total installed capacity of circa 12,000MW. This work has been done using the formulation of various cases of load dispatch equations relevant to the subject and hybrid computational approach i.e., Non-Linear Programming (NLP) optimization method. The authors have applied the various formulations to the operations information at Afam Power Plant. Results revealed that for load dispatch without power generator constraints, generated power is about 650MW, while for load dispatch with power generator constraints, generated power is about 610MW. The results also show that Afam power plant did not produce maximum output in most of the years between 2010 and 2016. The worst years were 2011, 2014 and 2016 due to obvious issues ranging from maintenance downtime to unavailability of transmission line to receive generated power. The research shows that the output of Afam power plant started to improve from 2017 to first Quarter of 2020. Further analysis shows that for system losses and generator limits, dispatch and total cost in naira per hour for a given number of generators load demand can be determined.

KEYWORDS: Afam power plant, Effective load dispatch, Energy, Equations, Generation, Optimization, Output, Programming.

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1.0 INTRODUCTION

Electrification utility is targeted at making available better quality and dependable electrical generation to consumers at an affordable cost while meeting the operating limits of generation equipment (Ahiakwo, 2019). The operator communicates the economic load dispatch

problem that determines the best combination of the generated output of all online generators to reduce the overall fuel cost. Some classical algorithms like Newton methods, lambda iteration and gradient method would have solved economic load dispatch problems if the curves in fuel-cost of the generation units are piecewise linear and monotonically growing. But, in the real sense of it, the input-output characteristics with respect to the generation units are non-linear, discrete and non-smooth in nature leading to prohibitive ramp rate limits, multi fuel effects and operating zones (Alawode, 2011).

Power system optimization came into focus as a result of developments in computing and optimization theories. In the early 20th century, optimal power flow problems were resolved by engineers and technical operators with in-depth experience using judgment, outdated, and primitive tools, which include specialized rules and analog network analyzer. Later, computational devices were introduced to help experienced operators.

In power systems, three types of problems commonly encountered include:

- (i) Economic dispatch
- (ii) Load flow (power flow)
- (iii) Optimal Power flow

Economic dispatch problem analyzes and describes different formulation to determine and find out the least-cost generation dispatch to enable a given load to be served with a reserve margin. These formulations most times, do not use power flow constraints. The load flow problem refers to load transmission and load network equations. Load flow (power flow) techniques are mathematical models which are not feasible optimal solutions. Present Power flow equations do not take note of generator reactive

power limits or transmission line limits. These constraints are written and programmed into one of many available power flow softwares. The third type (the optimal power flow) finds the optimal solution to a given objective function subject to the power flow constraints and many other constraints which include transmission stability, voltage constraints, generator minimum output constraints as well as limits on switching mechanical equipment. Optimal Power Flow (OPF) is also known as Security-Constrained Economic Dispatch (SCED). There are several formulations with different constraints, objective functions and solution techniques that have been seen as optimal power flow.

Huneault and Galliana (1994) gave an extensive analysis of optimal power flow problems with over 300 articles surveyed. After citing over 200 articles, they concluded that the Optimal Power Flow (OPF) history was characterized as the application of ever-increasing powerful optimization tools to a problem. Their paper outlined OPF evolutions grouped by various solution methods. According to them, the various solution methods included gradients techniques, quadratic programming, penalty techniques and linear programming. The authors further asserted that the OPF was still a complex mathematical problem because the algorithms could not compute quickly as required and they were prone to serious errors and convergence problems.

According to Muiler (1998), researchers later in the course of their various studies identified challenges associated with solving the OPF. These challenges include modeling discrete variables, computing time, solution reliability, local minima and lack of uniform problem definition. It was opined that today, with advances in Mixed Integer Programming (MIP), discrete variables could therefore be timely modeled.

The most comprehensive survey was put forward on optimal power dispatch and as a result of IEEE research group presented a bibliography analysis and survey of major economic security function in 1981 (Khamees *et al.*, 2016). Several surveys

were carried out on economic dispatch method (Chowdhurry, 1999). In the quest to put forward an efficient workable methodology, a review of some selected optimal power flow techniques was presented. It was asserted that the solution methodologies of OPF problems can be broadly grouped into two major categories: classical (conventional) methods and intelligent methods. He further analyzed an in-depth sub-division of these categories in a tree diagram as shown in Fig 1.

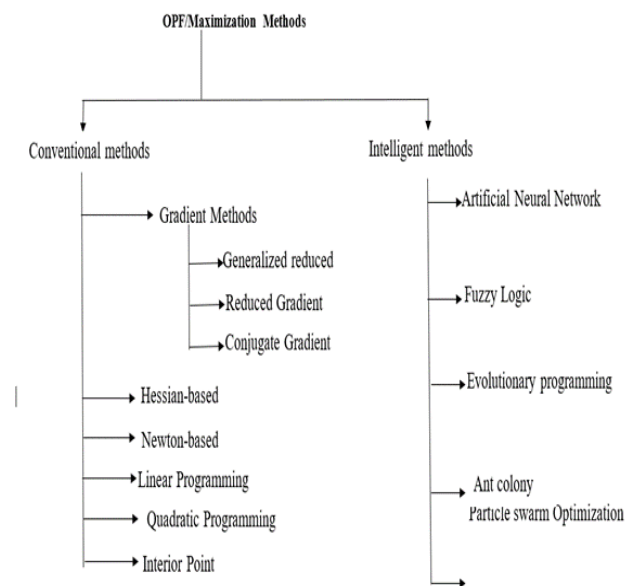


Figure 1 OPF Tree Diagram

2.0 MATERIALS AND METHODS

2.1 Effective Load Dispatch Cases

Case 1: Optimal Dispatch and Total Cost

Optimal dispatch and total cost are calculated as:

$$F_i = C_i + b_i P_i + a_i P_i^2 \quad (1)$$

If $i = 1, 2, 3, 4, \dots, N$ number of generators:

$$F_1 = C_1 + b_1 P_1 + a_1 P_1^2 \quad (2)$$

Similarly,

$$F_2 = C_2 + b_2 P_2 + a_2 P_2^2 \quad (3)$$

$$F_3 = C_3 + b_3 P_3 + a_3 P_3^2 \quad (4)$$

$$F_4 = C_4 + b_4 P_4 + a_4 P_4^2 \quad (5)$$

$$F_N = C_N + b_N P_N + a_N P_N^2 \quad (6)$$

Considering the system losses and generator limits, dispatch and total cost in N/hr for N-generator for the given load demand can be determined.

Case 2: Output Maximization and Operational Cost

The major cost of plant operation is fuel. The fuel cost curve is assumed to be parabolic of the form,

$$C_i = a_i + b_i P_i + C_i P_i^2 \quad (7)$$

In this case, the incremental fuel - cost curve (slope) of the fuel cost curve given as:

$$\frac{dc_i}{dp_i} = 2C_i P_i + b_i \quad (8)$$

Case 3: To Optimize the Total Cost of Generation

Supplying the given load demand, P_D given as:

$$C_t = \sum_{i=1}^{ng} C_i \quad (9)$$

Where:

$$C_i = a_i + b_i P_i + C_i P_i^2 \quad (10)$$

Then;

$$C_t = \sum_{i=1}^n (a_i + b_i P_i + C_i P_i^2) \quad (11)$$

Subject to constraint, given as:

$$\sum_{i=1}^{ng} P_i = P_D \quad (12)$$

If the number of generators is ng while total number of generators is n ; this can be represented as:

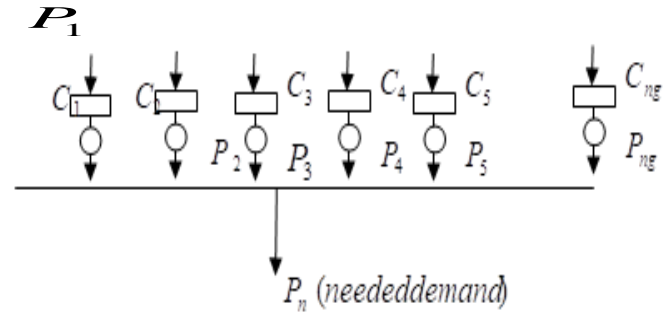


Figure 2 Cost Optimization of Connected Generators

The minimal incremental function (λ) can be represented as:

$$\mu = C_t + \lambda \left(P_D - \sum_{i=1}^{ng} P_i \right) \quad (13)$$

Obeying the necessary condition for minimization given as:

$$\frac{\partial \mu}{\partial p_i} = 0 \quad (14)$$

$$\frac{\partial \mu}{\partial \lambda} = 0 \quad (15)$$

The first condition given as:

$$\frac{\partial c_i}{\partial p_i} + \lambda (0 - 1) = 0 \quad (16)$$

But,

$$C_t = C_2 + C_3 + \dots + C_{ng}$$

Therefore,

$$\frac{\partial c_i}{\partial p_i} = \frac{dc_i}{dp_i} = \lambda \quad (17)$$

Thus, the condition for optimal dispatch is given as:



$$\frac{dc_i}{dp_i} = \lambda, i=1,2,\dots,ng \quad (18)$$

Thus, rewriting (18) gives:

$$C_t = \sum_{i=1}^{ng} C_i \quad (19)$$

This can be represented as:

$$C_t = \sum_{i=1}^{ng} (a_i + b_i P_i + C_i P_i^2) \quad (20)$$

Differentiating (20) gives:

$$\frac{\partial c_i}{\partial p_i} = 2C_i P_i + b_i \quad (21)$$

$$\lambda = 2C_i P_i + b_i \quad (22)$$

Rearranging (22) gives:

$$\sum_{i=1}^{ng} \frac{\lambda - b_i}{2C_i} = P_i \quad (23)$$

Similarly,

$$\sum_{i=1}^{ng} \frac{\lambda - b_i}{2C_i} = P_D \quad (24)$$

Solving for λ , we have;

$$\sum_{i=1}^{ng} \left(\frac{\lambda}{2C_i} - \frac{b_i}{2C_i} \right) = P_D \quad (25)$$

Or

$$b_i + 2C_i P_i = \lambda \quad (26)$$

The second condition states:

$$\sum_{i=1}^{ng} P_i = P_D \text{ (which is the constraint condition)}$$

Therefore, for (λ) optional generation, it is stated as;

$$P_i = \frac{\lambda - b_i}{2C_i} \quad (27)$$

Thus,

$$\sum_{i=1}^{ng} \frac{\lambda - b_i}{2C_i} = P_D \quad (28)$$

Hence, λ -solving for optimal generation is given as:

$$\lambda = P_D + \frac{\sum_{i=1}^{ng} \frac{b_i}{2C_i}}{\sum_{i=1}^{ng} \frac{1}{2C_i}} \quad (29)$$

However, optimizing by iteration of (29) gives:

$$f(\lambda) = \sum_{i=1}^{ng} \frac{\lambda - b_i}{2C_i} = P_D \quad (30)$$

Making use of the first order expressions by Taylor's series expansion around point $\lambda^{(k)}$ gives;

$$f(\lambda)^k + \left(\frac{df(\lambda)}{d\lambda} \right)^{(k)} D\lambda^{(k)} = P_D \quad (31)$$

But

$$\sum_{i=1}^{ng} (Y_{2C_i}) = P_D + \sum_{i=1}^{ng} \frac{b_i}{2C_i} \quad (32)$$

Or

$$\lambda \sum_{i=1}^{ng} \left(\frac{1}{2} C_i \right) = P_D + \sum_{i=1}^{ng} \frac{b_i}{2C_i} \quad (33)$$

$$\lambda = P_D + \frac{\sum_{i=1}^{ng} b_i / 2C_i}{\sum_{i=1}^{ng} (1/2C_i)} \quad (34)$$

Hence;

$$D\lambda^{(k)} = \frac{P_D - f(\lambda)^k}{\left(\frac{df(\lambda)}{d\lambda} \right)^{(k)}} = \frac{P_D - \sum_{i=1}^{ng} P_i^k}{\left(\frac{df(\lambda)}{d\lambda} \right)^{(k)}} \quad (35)$$

Or

$$= \frac{\Delta P^{(k)}}{\left(\frac{df(\lambda)}{d\lambda} \right)^{(k)}} \quad (36)$$



$$= \frac{\Delta P^{(k)}}{\sum (dp_i/d\lambda)^k}$$

(37)

Case 4: Transmission Loss Equation

The partial derivative ($\frac{rP_L}{rPG_i}$) is regarded as

Incremental Transmission Loss (ITL) related to the generating plant.

There are several approaches used to design a transmission loss model, which is related to transmission loss through the means of generation energy through its B- coefficients, also used in loss management with respect to profitability planning between plant loads. Loss formula through B-coefficients is generally given as;

$$P_L = \sum_{i=1}^N \sum_{j=1}^N PG_i B_{G_i} + \sum_{j=1}^N B_{io} PG_i + B_{oo} \quad (38)$$

Case 5: Optimal Schedule for Output Maximization (Afam Generating Station)

Factors influencing the minimum cost of power generation are as follow:

- i. Operating efficiency of prime mover
- ii. Fuel costs
- iii. Transmission losses

Case 6: Economic Dispatch and Generator Limits

Economic load dispatch regarding generator limit can be done by;

- i. Minimizing the objective or cost function overall plant
- ii. Use of a quadratic polynomial cost function for each plant:

$$C_{total} = \sum_{i=1}^{ng} a_i + b_i P_i + C_i P_i^2 \quad (39)$$

- iii. using total demand being equal to the sum of generators' output: equality constraint

That is,

$$\sum_{i=1}^{ng} P_i = P_{Demand} \quad (40)$$

Case 7: Incremental Loss (λ) for Generating Plants ($n = 1,2,3....ng$)

Plant 1: $\frac{dc_1}{dp_1} (N/MWh)$

Plant 2: $\frac{dc_2}{dp_2} (N/MWh)$

Plant 3: $\frac{dc_3}{dp_3} (N/MWh)$

Plant 4: $\frac{dc_4}{dp_4} (N/MWh)$

Plant 5: $\frac{dc_5}{dp_5} (N/MWh)$

Plant 6: $\frac{dc_6}{dp_6} (N/MWh)$

Plant N: $\frac{dc_N}{dp_N} (MW)$

Case 8: Plant Coefficients

$$C_i = a_i + b_i P_i + C_i P_i^2$$

$$C_{eff} \text{ of plant capacity} = \begin{bmatrix} a_1 b_1 C_1 \\ a_2 b_2 C_2 \\ a_3 b_3 C_3 \\ a_4 b_4 C_4 \\ a_5 b_5 C_5 \\ a_6 b_6 C_6 \end{bmatrix} =$$

0.22	58	1.14
0.17	74	0.72
0.05	69	1.8
0.06	69	2.2
0.04	69	3.15
0.04	70.5	2.70

Plant demand: MW

2.3 Generator Fuel Cost Model for Afam Power Plant

For Afam power plant generators configuration, optimization of the sum of all fuel cost, F_T , associated with the generators is expressed as follows:

$$F_T = F_1 + F_2 + \dots F_{NG} \quad (41)$$

Modifying (41),

$$F_T = \sum_{i=1}^n F_i(Pg_i) \quad (42)$$

Equation (42) is limited to power balance and unit generation limit. Fuel cost optimization in general could be modelled using Lagrange Multiplier analysis. This helps to convert a constraint problem into a problem that is unconstrained (Shunpike, 2018). The multiplication with Lagrange multiplier and the addition of constraint function to objective function is expressed as shown:

$$\rho(Pg_i, \lambda) = F_T + \lambda(P_j - \sum_{i=0}^{ng} Pg_i) \quad (43)$$

Where λ represent the Lagrange Multiplier. Then,

$$\frac{\partial \rho(Pg_i, \lambda)}{\partial Pg_i} = \frac{\partial F_T}{\partial Pg_i} - \lambda = 0 \quad (44)$$

Simplifying (47),

$$\frac{\partial \rho(Pg_i, \lambda)}{\partial \lambda} = P_d - \sum_{i=1}^{ng} Pg_i = 0 \quad (45)$$

From (44),

$$\frac{\partial F_T}{\partial Pg_i} = \lambda \quad i = 1, 2, 3 \dots Ng \quad (46)$$

Evaluating (42) and (46), we have:

$$\frac{\partial F_T}{\partial Pg_i} = \frac{\partial \sum_{i=1}^{Ng} Pg_i}{\partial Pg_i} = \lambda \quad (47)$$

Therefore,

$$\frac{\partial F_1(Pg_i)}{\partial Pg_i} = \frac{\partial F_2(Pg_2)}{\partial Pg_2} = \dots = \frac{\partial F_n(Pg_n)}{\partial Pg_n} = \lambda \quad (48)$$

Equation (48) expresses clearly that the optimal loading of generator is feasible at the point where the input-output slopes. Therefore, the characteristics of all associated generators are the same. In addition to this assertion, the optimal dispatch is equal to the point of same amount of incremental cost for all the system generators.

2.4 Power Loss Integration Optimization Model for Afam Power Plant

The model above did not consider transmission losses since transmission losses most times vary between 5 to 10 percent of the entire system load.

It is therefore unrealistic to neglect transmission losses. Modifying (41) further, the P_{loss} represents the active power loss in the entire system and is modelled using (43) as the generation limit:

$$\sum_{i=1}^{ng} Pg_i = P_d + P_{loss} \quad (49)$$

where P_d = power demand

The major portion of the operating cost is changing the associated transmission losses in a given generator with its percentage of fuel cost. It must be noted that the transmission losses in Afam power plant are analyzed as a function of associated generator power with respect to B-coefficient as shown:

$$P_{loss} = \sum_{i=1}^N \sum_{j=1}^N P_i B_{ij} P_j \quad (50)$$

Where N and B_{ij} represent the overall number of buses from the system and loss coefficient that exist between individual buses respectively.

P_j is the real power injected into various node buses.

Hence, applying the Lagrange multiplier to optimize the power loss,

$$\rho(Pg_i, \lambda) = F_T + \lambda(P_d + P_{loss} - \sum_{i=1}^{ng} Pg_i) \quad (51)$$

$$\frac{\partial \rho(Pg_i, \lambda)}{\partial Pg_i} = \frac{\partial F_T}{\partial Pg_i} - \lambda \left(\frac{\partial P_{loss}}{\partial Pg_i} - 1 \right) \quad (52)$$

$$\frac{\partial F_T}{\partial Pg_i} = \lambda \left(\frac{\partial P_{loss}}{\partial Pg_i} - 1 \right) \quad i=1, 2, \dots, ng \quad (53)$$

$$\frac{\partial \rho(Pg_i, \lambda)}{\partial \lambda} = P_d + P_{loss} - \sum_{i=1}^{ng} Pg_i = 0 \quad (54)$$

$\frac{\partial P_{loss}}{\partial Pg_i}$ represents the incremental transmission losses (Braide *et al.*, 2017).

3.0 RESULTS AND DISCUSSION

Raw operation data were obtained from Afam power plant as shown in the Table 1 which is for



January to December 2011. Table 2, on the other hand, shows computed operational data for the whole period under review i.e., 2011 to 2020. Non-Linear Programming (NLP) method was used during the computations.

Table 1 Computed Operational Data from Afam Power Plant Output for 2011.

Year	Month	Energy Generated (EG) (MWH)	Unit Price	Energy Charged (EC) (N)	Energy Charge Optimization (ECO) (N)	Total EG/Yr (MWH)	EC Yearly Total (N)	ECO Yearly Total (N)
2011	January	0	5102	0				
	February	0	5102	0				
	March	0	5102	0	0			
	April	0	5102	0	0			
	May	0	5102	0	0			
	June	0	5102	0	0			
	July	0	5102	0	0			
	August	0	5102	0	0			
	September	19002.90	5102	96952795.8	96952795.8			
	October	32835.00	5102	167524170	70571374.2			
	November	40213.00	5102	205166726	134595351.8			
	December	25271.40	5102	128934682.8	91292126.8	117322.3	598578374.6	393411648.6

Table 2 10-year Computed Operational Data from Afam Power Plant.

SN	YEAR	TEG/YEAR (MW)
1	2011	117,322.30
2	2012	382,724.10
3	2013	254,548.70
4	2014	267,062
5	2015	6,928
6	2016	0
7	2017	5,203.55
8	2018	10,111.89
9	2019	192,51.73
10	2020	10,680.73

Results and discussions from the analysis of Tables 1 & 2 are the same as for Figure 6. Deductions in this paper reveal that there are

various ways to theoretically evaluate a power plant with the aim of maximizing its output. In the same vein, the load dispatch capability of a power plant may also be evaluated using algorithms and equations with obvious consideration of various viable cases.

For system losses and generator limits, dispatch and total cost in naira per hour for N-generator for a given load demand can be determined. Cost of operating a power is majorly incident on the fuel. It was deduced that the fuel cost equation/curve is parabolic while the fuel-cost relation is somewhat linear as deduced in (11). In the various cases considered for the economic load dispatch derivations, it was observed that there is maximum output from the power plant if cost of fuel is minimized. The onus is now on power plant operators to ensure that all that is required are done during maintenance to reduce fuel consumption. This may be achieved with regular

inspection maintenance. Case 3 clearly reveals that needed demand (or output of a power plant) may be increased if the total cost for ‘n’ generators are optimized. This means that the overall output of a power plant is the sum total of the efficient or optimized output of individual generators. Furthermore, transmission losses were also considered and the deduced equation shows that such losses may be minimized by minimizing the B-coefficients of the transmission lines. The operations schedule applied by power generating plants also influences the efficiency of such power plant.

From (9) & (10) and Fig 3, when applied to Afam power plant in relation this research, the incremental fuel cost curve lies within the boundaries of lines ‘a’ and ‘b’. It is deduced from the results of the application of the equations that power output increases with increase intake of fuel which in turn means increase in cost. The efficiency of any power plant (and Afam is not left out) depends, however, on the total load demand handled at any point on the curve. At P_{gmin} fuel consumed is less than what is consumed at P_{gmax} . Curve b shows the trend for an optimal operation of the Afam power plant that will ensure maximization of output to the grid. The cost of fuel consumed is minimized when output power takes maximum load.

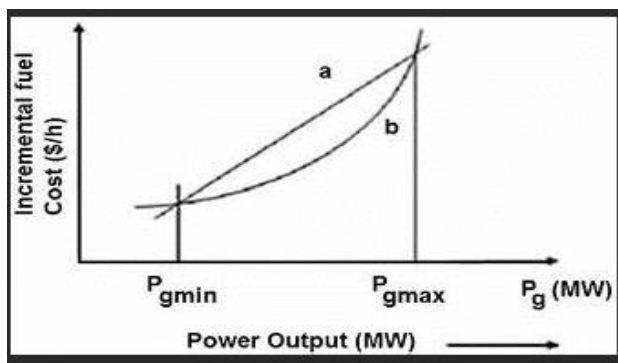


Figure 3 Incremental Cost Curve of Power Generation (Dhamanda *et al.*, 2013)

Furthermore, considering case 6 discussed above, economic load dispatch regarding generator limit gives different results when considered with constraints and without constraints. Applying the quadratic polynomial cost function for each plant

with a number of iterations, the following curves were derived.

For another analysis different from the above, considering three of the generators in the power plant, Figs 4 and 5 show that the ability to dispatch generated load is comparatively higher without constraints. It is observed that in Fig 4 (the case of load dispatch without power generator constraints), P_G is about 650MW, while in Fig 5 (the case of load dispatch with power generator constraints), P_G is about 610MW.

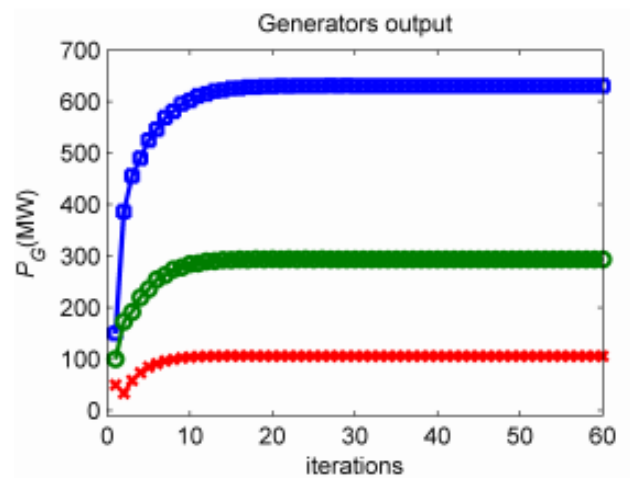


Figure 4 Load Dispatch without Power Generator Constraints

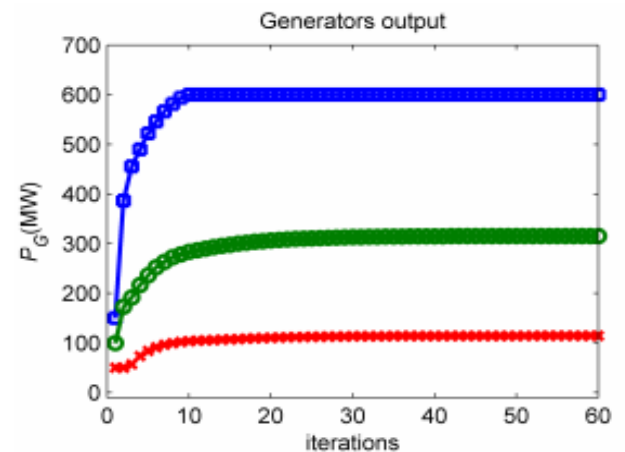


Figure 5 Load Dispatch with Power Generator Constraints

Considering the total generation at Afam within the period 2010 to 2020, Figure 6 shows a graphical result obtained. Afam power plant did not produce maximally in most of the years

between 2010 and 2016 (indicated by the purple trend/line on the graph). The worst years were 2011, 2014 and 2016 due to obvious issues ranging from maintenance downtime to unavailability of transmission line to receive generated power. The green trend/line is the optimised (maximized) output i.e., what the output should have been if the plant is maximized. The deep (indicated on the graph) experienced in 2016 was due to a complete outage of all Afam Gas turbine Generators. The plant was brought back on stream by the operators towards 2017 when power output was averagely increasing to where it was in the first quarter of 2020 when the plant was visited by the author.

- TEC is Total Energy Charges (purple line)
- OTEC is Optimised TEC (green line)

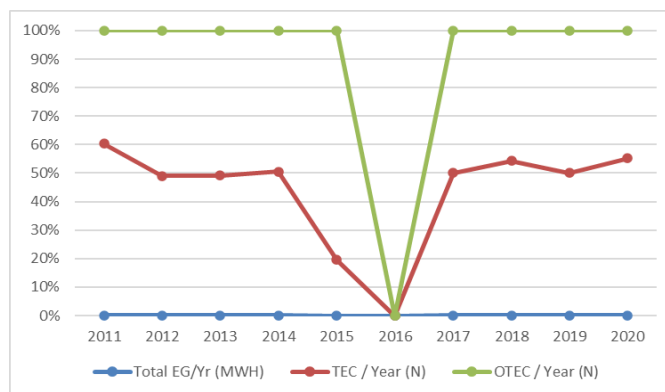


Figure 6 Graphical Representation of Percentage output of Afam Power Plant.

4.0 CONCLUSION

Models carried out in this report as applied to Afam power plant show that there is room for optimization at the plant. The fuel cost optimization model, when applied in real terms, is indicative that the total fuel consumed is not commensurate with the power output from the plant. The issue at Afam was, however, traced to more of fuel quality. Dirt and debris are common composition of fuel that is available to power plants in Nigeria and the APP is not left out, though its fuel is sourced mainly from Okoloma, Agbada and Obigbo gas plants belonging to SPDC. Poor quality of fuel leads to fouling of the internal parts of the core engine i.e., the compressor turbine and the combustion chambers.

Once that (fouling) happens over time, the output of the gas turbine drops, and the efficiency is reduced. Losses due to transmission lines were also modelled and results confirmed that transmission losses also weigh on Afam Power Plant. This happens with the outage of transmission lines used for the evacuation of generated power – 132kV and 330kV lines - as the case may be. Evacuation line outages could be due to line faults, switchgear earth-faults, and earth-leakages. These force the operators to shut-down the Gas Turbine Generators since there is nowhere to evacuate generated power to.

The results validate the equations and formulations in this paper, as applied to Afam power plant in the output of the generators during operations. Findings from the research will help in predicting the plant operating envelope based on data acquired from the plant operators. This knowledge provides reference data sets and graphical trend analysis which are veritable tools for plant analysis.

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